JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 17, No. 5, September–October 1994

Controls Design Challenge: A Variable Structure Approach

Ajay Thukral* and Mario Innocenti†
Auburn University, Auburn, Alabama 36849

The purpose of this paper is to investigate variable structure control as a tool for the design of flight control systems. Variable structure control allows the direct incorporation of model uncertainties and nonlinearities in the design process, yielding a piecewise continuous structure for the compensator that maintains the system dynamics along specified stable trajectories (sliding hypersurfaces). Under mild restrictions, the resulting control law is robust with respect to unknown but bounded parameter variations and other uncertainties. A linearized model of an aircraft is used for the design, and a flight control system is synthesized, which gives desired response to initial conditions and tracks a commanded altitude. The control synthesis and simulation are performed using computer-aided software on a personal computer.

I. Introduction

THE purpose of the paper is to evaluate the capabilities of variable structure systems (VSS) in designing a flight control system within the framework of the controls design challenge.[1] The work presented here is preliminary in nature, and its purpose is to illustrate the main ideas behind VSS control, its potential application to aircraft control related problems, and the ease with which preliminary flight control synthesis can be performed on inexpensive computer work stations.

Variable structure control has been described in the Soviet literature since the early 1960s in the work of Emel'yanov,[2] Utkin,[3] and Itkis,[4] among others. Invariance of VSS to a class of disturbances and parameter variations was first developed by Drazenovic in 1969.[5] In the past two decades, much research has been performed by the international community in this area linking VSS to adaptive control and model reference adaptive control, using Lyapunov control techniques, deriving connections with hyperstability theory, and solving VSS tracking problems (see Ref. 6 for a survey on the subject). Most of the applications of VSS have been in the areas of industrial and robot manipulator control. Recently some work has been done in the aerospace field; applications to aircraft control have been presented by Calise and Kramer[7] where robustness with respect to nonlinearities was addressed, by Mudge and Patton[8] who solved the sensitivity to parameter variations by incorporating eigenstructure assignment in the structure of the control law, by Ref. 9 where Slotine's concept of boundary layer was used to eliminate "chattering." Variable Structure Systems and Lyapunov stability theory were also used by Vadali[10] in designing a large-angle maneuvers controller for a spacecraft.

The essential feature of a variable structure controller is the nonlinear feedback control with discontinuity on one or more manifolds (sliding hyperplanes) in the state space, or error space in the case of model following control. This type of methodology is attractive in the control design of nonlinear uncertain dynamic systems with uncertainties and nonlinearities of unknown structure as long as they are bounded and occurring within a subspace of the state space.[4] Ryan and Corless[11] have also shown that VSS could be used to establish "almost certain" convergence to a vicinity of the origin for a class of uncertain systems.

The extended flight envelope of recent aerospace systems is posing a challenge to the control system designer who is faced with highly nonlinear uncertain systems whose control could be addressed by employing VSS techniques.

Received Dec. 6, 1991; revision received July 19, 1993; accepted for publication July 20, 1993.

*Ph.D. Student. Student Member AIAA.

†Associate Professor, Department of Aerospace Engineering; currently with the Department of Electrical Systems and Automation, University of Pisa, Italy. Senior Member AIAA.

II. Variable Structure Systems Control

A brief description of the principles of variable structure systems is now presented, which follows essentially those described in Refs. 3 and 6. The basic feature of VSS is sliding motion. This occurs when the system state continuously crosses a switching manifold because all motion in its vicinity is directed inward. When the motion occurs in all the switching surfaces, the system is said to be in the sliding mode, and it is equivalent to an unforced system of lower order. The design of a variable structure controller consists of several steps: 1) the choice of switching surfaces; 2) the determination of the control law; and 3) the switching logic associated with the discontinuity surfaces (usually fixed hyperplanes through the origin of the state space). To ensure that the state reaches the origin along the sliding trajectories, the equivalent system must be asymptotically stable and this defines the selection of the switching hyperplanes (sometimes called the "existence" problem), which is completely independent of the choice of control laws. The selection of the control law is the so-called "reachability" problem and it requires the reaching of the sliding mode from some initial state.

During sliding, the discontinuous control chatters about the switching surface at high frequency. Chatter is perhaps the major problem associated with this type of control; it can require high energy effort from the actuators, thus leading to continuous saturation, and it can excite neglected high-order dynamics. This is perhaps the major reason why VSS has not yet found wider acceptance in the flight control community, where smoothness of actuation is desirable, and saturation may lead to such instabilities as pilot-induced oscillations.

There are several ways to mitigate the effects of chattering with little loss in performance, such as the definition of a boundary layer near the sliding surface, as introduced by Slotine,[9] or the introduction of a smoothing parameter in a unit vector-type control law, as shown by Ambrosino et al.,[12] Burton and Zinober,[13] and Balestrino et al.[14] The latter method is used in this paper.

The general control problem can be stated as follows: given the uncertain controllable dynamic system of the form

$$\dot{x} = (A + \Delta A)x + (B + \Delta B)u + Cv, \qquad y = x + w \tag{1}$$

where the state and input vectors have dimensions n and m respectively, $v(t)$ is a one-dimensional disturbance vector also representing nonlinearities, and $w(t)$ is a vector of output (measurement) uncertainties. The parameter variation matrices ΔA and ΔB can be uncertain and time varying. Matching conditions are assumed to be satisfied by the matrices ΔA, ΔB, and C, thus satisfying Drazenovic invariance conditions as well as perfect model following.[15] Because matching requires ΔA, ΔB, and C to be in the range space of B (assumed to be full rank), the following relations are valid:

$$\Delta A = BD, \qquad \Delta B = BE, \qquad C = BF \tag{2}$$

where D, E, and F have dimension $n \times n$, $m \times m$, and $m \times l$, respectively. The purpose of a VSS design is to determine the control law u such that

$$u_i(x) = \begin{cases} u_i^+ & \text{for } s_i(x) > 0 \\ u_i^- & \text{for } s_i(x) < 0 \end{cases} \tag{3}$$

The switching hyperplanes are denoted in matrix form by

$$s = Gx \tag{4}$$

with s m-dimensional and G an $m \times n$ constant matrix. For a stable sliding motion to occur on all surfaces, the following conditions must be satisfied based on Lyapunov's stability theory:

$$\begin{gathered} s_i \dot{s}_i < 0 \text{ in the vicinity of } s_i = 0 \\ s = Gx = \dot{s} = G\dot{x} = 0 \text{ in the sliding mode} \end{gathered} \tag{5}$$

Because the sliding mode belongs to the null space of G, if the product GB is nonsingular, the sliding motion is independent of the control law. During sliding, from Eqs. (5) and (1) we can, therefore, determine an equivalent control law as

$$\begin{gathered} u = -(GB)^{-1}G[Ax + h] \\ h = \Delta Ax + \Delta Bu + Cv \end{gathered} \tag{6}$$

and because the matching conditions (2) are assumed to be valid, the system dynamics during sliding are governed by

$$\dot{x} = \left[I - B(GB)^{-1}G\right]Ax \tag{7}$$

showing the sliding motion to be insensitive to unknown but bounded parameter variations and disturbances. The selection of the switching surfaces; i.e., G, then depends on the desired system behavior during sliding, and given by Eq. (7).

To select the switching surfaces we consider a nominal system extracted from Eq. (1) and given by

$$\dot{x} = Ax + Bu, \qquad y = x, \qquad s = Gx \tag{8}$$

To simplify the design scheme, we can transform Eq. (8) into a controllable canonical form using the transformation $q = Tx$, where T is an orthogonal matrix. Then Eq. (8) becomes

$$\begin{gathered} \dot{q} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} q + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u \\ y = T^T q, \qquad s = GT^T q = [G_1 \quad G_2]q \end{gathered} \tag{9}$$

Note that because GB is nonsingular, so is G_2B_2. During sliding, we have the following, from $s = 0$:

$$\begin{gathered} \dot{q}_1 = [A_{11} - A_{12}K]q_1 \\ q_2 = -Kq_1 \end{gathered}$$

with

$$K = G_2^{-1}G_1 \tag{10}$$

The sliding motion occurs, therefore, in the n–m dimensional subspace of the state space. The choice of K and consequently of G is free for the designer; several methods have been used in the literature, such as pole placement, eigenstructure assignment,[8] and optimal control.[16] Using the latter method to find K, we can set up a linear-quadratic-regulator (LQR) synthesis to minimize

$$J = \frac{1}{2}\int_t^\infty [x^T Q_x]\,dt \text{ with } Q > 0 \tag{10'}$$

subject to the constraints given by Eq. (10). This index of performance can be reduced to the transformed state space q by using T. We can write

$$TQT^T = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}$$

if we let

$$\begin{gathered} Q^* = Q_{11} - Q_{12}Q_{22}^{-1}Q_{21} \\ A^* = A_{11} - A_{12}Q_{22}^{-1}Q_{21} \\ \zeta = q_2 + Q_{22}^{-1}Q_{21}q_1 \end{gathered} \tag{11}$$

the LQR problem has now the following standard form:

$$\begin{gathered} J = \frac{1}{2}\int_t^\infty \left[q_1^T Q^* q_1 + \zeta^T Q_{22}\zeta\right] dt \\ \dot{q}_1 = A^* q_1 + A_{12}\zeta \end{gathered} \tag{12}$$

After solving for the appropriate Riccati matrix P from Eq. (12), we obtain

$$K = Q_{22}^{-1}\left[Q_{21} + A_{12}^T P\right] \tag{13}$$

A simple method for deriving the switching matrix G in Eq. (10) from Eq. (13) is given in Ref. 17. If we let $G_2 = I_m$, then $[G_1 \ G_2] = GT^T = [K \ I_m]$, thus giving the following:

$$G = [K \quad I_m]^T \tag{14}$$

Having specified the sliding trajectories, we now turn our attention to the computation of the control law u, which will drive the state vector $\boldsymbol{x}$ into the null space of G and maintain it there. The choice of control is only limited by the discontinuity on one or more subspaces containing the null space of G as stated in Eq. (3).

In general, the VSS control law u consists of a linear component u^L and a nonlinear one u^N combined together to produce the feedback, with the nonlinear component incorporating the discontinuous elements. In the present work, the following structure for the control law has been chosen:

$$u = u^L + u^N = Lx + \rho\frac{Nx}{\|Mx\|} \tag{15}$$

The linear component is a typical full state feedback, whereas the nonlinear element has a unit vector form[11,15] easier to implement than other structures. The parameter ρ is free to be chosen and the matrices N, M, and G belong to the same null space.

To compute the gain matrices L, M, and N in Eq. (15) we follow the procedure described in Refs. 6 and 17. Let us define a new transformation matrix T_2 as

$$T_2 = \begin{bmatrix} I_{n-m} & 0 \\ K & I_m \end{bmatrix}$$

Using the preceding matrix, the state vector $\boldsymbol{q}$ is changed into $z = T_2 q$ with $z_1 = q_1$, and $z_2 = Kq_1 + q_2$. The dynamics of z are then given by

$$\begin{gathered} \dot{z}_1 = \Lambda_1 z_1 + A_{12}z_2 \\ \dot{z}_2 = \Lambda_2 z_1 + \Lambda_3 z_2 + B_2 u \end{gathered} \tag{16}$$

where

$$\begin{gathered} \Lambda_1 = A_{11} - A_{12}K \\ \Lambda_2 = K\Lambda_1 + A_{21} - A_{22}K \\ \Lambda_3 = KA_{12} + A_{22} \end{gathered} \tag{17}$$

To attain a sliding mode it is required from Eqs. (10) and (16) that $z = \dot{z} = 0$, therefore we can define $u(z) = u^L + u^N$ where

$$u^L(z) = -B_2^{-1}\left[\Lambda_2\left(\Lambda_3 - \Lambda_3^*\right)\right]z = -\Theta z \tag{18}$$

The stability matrix Λ_3^* has eigenvalues that determine the speed and transient characteristics with which the state vector asymptotically attains a sliding motion. The nonlinear component allows the state z_2 to reach the sliding mode in finite time. By defining $P_1 > 0$ solution of the Lyapunov equation $P_1\Lambda_3^* + (\Lambda_3^*)^T P_1 + I_m = 0$, we can set

$$u^N = -\rho\frac{B_2^{-1}P_1 z_2}{\|P_1 z_2\|} \tag{19}$$

Finally, returning to the original state vector $\boldsymbol{x}$, we have the control law given by Eq. (15) with gain matrices as follows:

$$L = -\Theta T_2 T$$
$$N = -B_2^{-1}[0\ P_1]T_2 T \tag{20}$$
$$M = [0\ P_1]T_2 T$$

When there are disturbances and parameter variations included in the system dynamics, as in Eq. (1), the control law (15) and gain matrices (20) still hold; however, the output vector y now appears in the control structure in place of x and ρ becomes a function of the off-nominal components ΔA, ΔB, etc. Details on the computation of ρ can be found in Refs. 11 and 18. Briefly, recalling the uncertain system model (1), and using Eqs. (2) and (15) we obtain the following form:

$$\dot{x} = (A + BL)x + Bu^N + Be$$
$$e = Dx + Lw + ELy + eu^N + Fv \tag{21}$$
$$u = Ly + \rho\frac{Ny}{\|Ny\|}$$

with the assumption that $[1 - \max\|E\|] > 0$, it can be shown that

$$\rho = A + B\|x\|$$
$$A = [1 - \max\|E\|]^{-1}\{[2\max\|D\| + \max\|L\| + \max\|EL\|]\rho_w + \max\|Fv\|\} \tag{22}$$
$$B = [1 - \max\|E\|]^{-1}\{\max\|D\| + \max\|L\|\}$$

and $\rho_w = \max\|w\|$.[18]

The procedure described in the preceding paragraphs is designed to drive and to maintain the system state into an ideal sliding mode. In practice, this is not achievable, and the state vector keeps crossing the sliding manifold at high frequency (chattering). The simplest way to overcome this problem is to "smooth" the discontinuous part of the control law u^N by substituting it with a continuous approximation. This is accomplished during the implementation of the control given in Eq. (22) by changing u to become

$$u = Ly + \rho\frac{Ny}{\|Ny\| + \delta} \tag{23}$$

where $\delta > 0$ is a small scalar. Although not rigorous (for a more formal approach see Ref. 11), the method has proved to be successful.

In summary, the VSS control design consists of the following steps:

1) Select sliding subspace using Eqs. (13) and (14).
2) Choose a control strategy of the form in Eq. (15).
3) Select the speed with which sliding is to be attained by choosing Λ_3^* in Eq. (18).
4) Compute the control gain matrices using Eq. (20).
5) Select ρ according to the perturbations included in the model, or else choose it to be a constant.
6) Implement a smoothed control law by proper choice of δ in Eq. (23).

In the present work, MATLAB software has been used to determine the control law. A macro (mfile) was written, which allowed interactive operation during the first four steps. The simulation used the block diagram graphic capabilities of SIMULINK and allowed to interactively, solve steps 5 and 6. In the next sections, some results relative to design challenge problem are presented limited to a linearized version of the equations of motion.

III. Aircraft Model Dynamics

For the purpose of preliminary validation, a linearized set of equations of motion was derived from the information provided by NASA. The system matrices are relative to a flight condition of Mach $M_{ref} = 0.6$ and altitude $h_{ref} = 39{,}800$ ft. The aircraft longitudinal motion is considered given by

$$\dot{x} = A_{LO}x + B_{LO}u \tag{24}$$

where $\boldsymbol{x} = [u, a, q, \theta, h]^T$ is the state vector with conventional state variables, and $\boldsymbol{u} = [\delta_E, \delta_{TH}]^T$ is the input vector consisting of elevator and throttle. The units for the variables are radians, feet, and seconds, as appropriate. Numerical values for A_{LO} and B_{LO} are given in the Appendix. Several simplifications were made in deriving Eq. (24): 1) the state vector is available for feedback; 2) no cross-coupling between longitudinal and lateral motions; 3) sensor dynamics as well as differential thrust were neglected; and 4) full afterburner and full power authority are available. In addition to Eq. (24), the engines and elevator actuators are modeled by first-order systems with break frequency of 20 rad/s. The engines lag is assumed to be equal to one s. In state space form, the actuator dynamics and engines lag are given by

$$\dot{r} = A_{ACT}r + B_{ACT}u_C, \qquad u = C_{ACT}r \tag{25}$$

where $r = [\delta_E \quad \delta_{TH} \quad \dot{\delta}_{TH}]^T$ and the commanded input vector is $u_C = [\delta_{EC} \quad \delta_{THC}]^T$. The system matrices in Eq. (25) are given in the appendix. Combining Eqs. (24) and (25), we get the augmented aircraft dynamics;

$$\dot{\chi} = \begin{bmatrix} A_{LO} & B_{LO}C_{ACT} \\ 0 & A_{ACT} \end{bmatrix}\chi + \begin{bmatrix} 0 \\ B_{ACT} \end{bmatrix}u_C = A_A\chi + B_A u_C$$
$$y = \chi \tag{26}$$

During the simulation, a saturation element was introduced in the elevator channel to limit displacement to +15/−25 deg. System (26) is the nominal system used to derive the control laws and corresponds to Eq. (8) of the previous section. Control laws are presented for initial condition response and for tracking of a specified altitude command. The controlled system behavior to sample uncertainties is also presented.

IV. Example 1: Initial Condition Response

This example considers the system response to an initial condition on altitude $h_0 = 10$ ft. The main objectives are to improve the damping of the natural modes, to evaluate the effect of smoothing on chattering, and to compare the control law with and without the nonlinear component. The control law (23) was computed with the following numerical values: $Q = 18$, $\rho = 1$, $\delta = 0.5$, and $\Lambda_3^* = \text{diag}[-.5, -1]$ and the resulting gain matrices L, M, and N can be found in the Appendix.

The chattering nature of the control, after the sliding mode is reached ($t = .75$ s), is evident in Figs. 1 and 2 showing commanded elevator and throttle. The figures also show how smoothing can effectively eliminate the chattering. Figure 3 shows the actual control inputs with elevator saturation during transient. Figures 4 and 5 show the altitude and speed responses respectively. The system's behavior with only the linear component of the control law is also shown

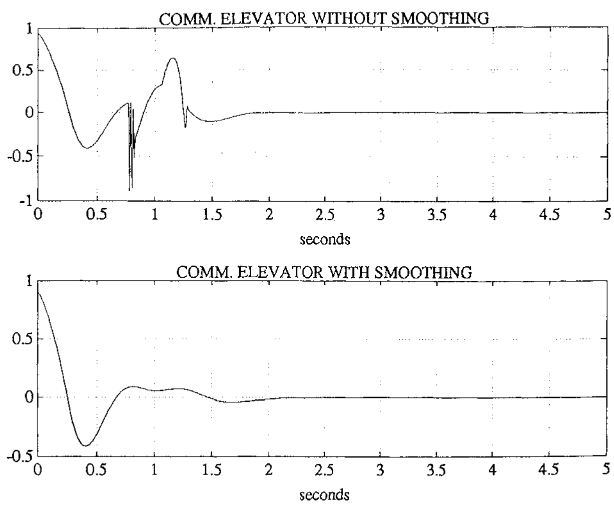

Fig. 1 Initial response commanded elevator.

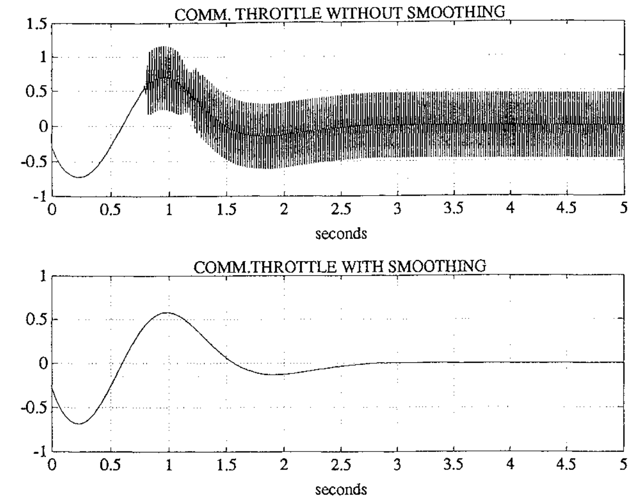

Fig. 2 Initial response commanded throttle.

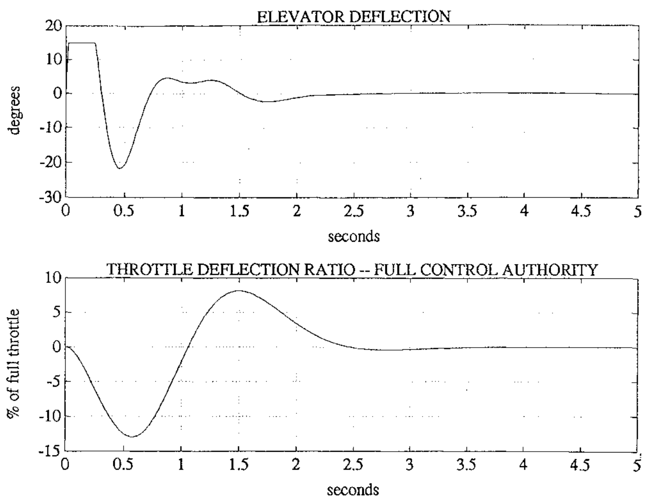

Fig. 3 Initial response control activity.

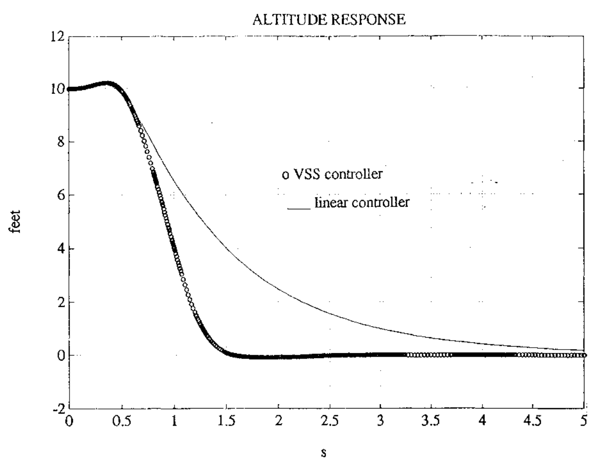

Fig. 4 Initial response altitude behavior.

clearly indicating the asymptotic character of the equivalent control and the necessity of having the nonlinear component to reach the zero state in a finite time. 2

V. Example 2: Command Tracking

This section considers the problem of tracking a commanded altitude ramp of 50 ft/s for 4 s, and it could be representative of an altitude following system. Tracking requirements include a maximum altitude error of +50 ft with almost constant speed (only a 1% in Mach number change is allowed).

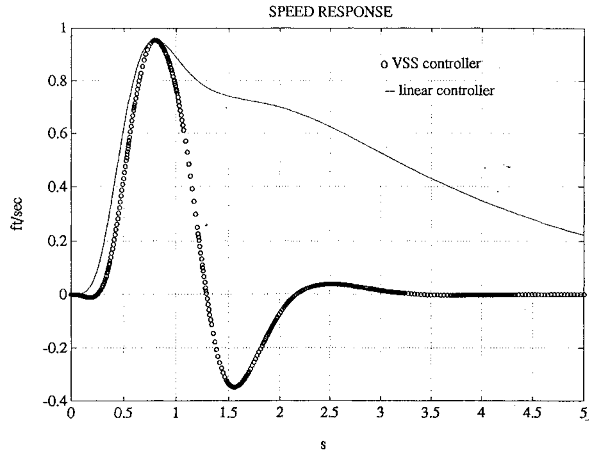

Fig. 5 Initial response speed behavior.

The problem of tracking is usually viewed in terms of model following, and a large literature exists addressing adaptive model following design using variable structure systems.[6] The procedure described in Sec. II is essentially carried over in the error space, once a suitable desired model behavior has been chosen. Here, we proceed along a simpler line by designing a feedforward prefilter based on Broussard's command generator tracker[19] and using the linear component of the VSS controller.

If we define the tracked state variable $h = H_\chi$, where χ is solution of Eq. (26) and the tracking command h_C output of the ramp generator, the feedforward gain matrix F_F is given by

$$F_F = [\Omega_{22} - L\Omega_{12}] \tag{27}$$

with

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} = \begin{bmatrix} A_A & B_A \\ H & 0 \end{bmatrix}^{-1}$$

The control law is computed with $Q = 18$, $\rho = 10$, $\delta = 0.5$, and $\Lambda_3^* = \text{diag}[-3, \ -3]$, and these are again found in the appendix. The block diagram of the closed-loop system and the controller used are shown in Figs. 6 and 7, respectively. Figure 8 shows altitude response and error, which remains within the required limit. Forward speed and Mach number perturbations are shown in Fig. 9, with the speed remaining well within the ± 10 ft/s requirement. Elevator and throttle activities are in Fig. 10. Although the throttle never exceeds 30% of full power, the elevator saturates as soon as the command changes slope (note that no rate limits were present in the simulation), and then goes back within the limits, thus illustrating the high gain characteristics of variable structure controller. It must be mentioned at this point that no particular fine tuning was performed in selecting Q, ρ, δ, and Λ_3^* except for achieving the required objectives.

VI. Example 3: Tracking in the Presence of Uncertainties

The problem of control in the presence of uncertainties, nonlinearities, parameter variations, and other unmodeled quantities has always been a central one in the design of feedback systems. Because an accurate analytical model of the aircraft was not yet available at the time the work was performed, the following analysis should be considered more as a qualitative validation rather than a final quantitative result.

The design parameters unchanged from the previous cases are $Q = 18$, $\rho = 10$, $\delta = 0.5$. The matrix Λ_3^* changes slightly because of stability limitations encountered because of the high speed required to attain the sliding mode (governed by the eigenvalues of Λ_3^*). Speed of response and bandwidth requirements always conflict

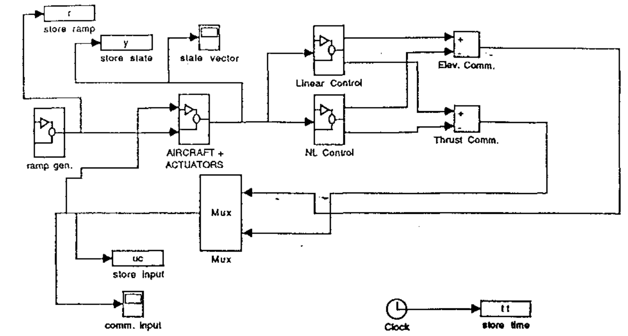

Fig. 6 Block diagram for altitude tracking simulation.

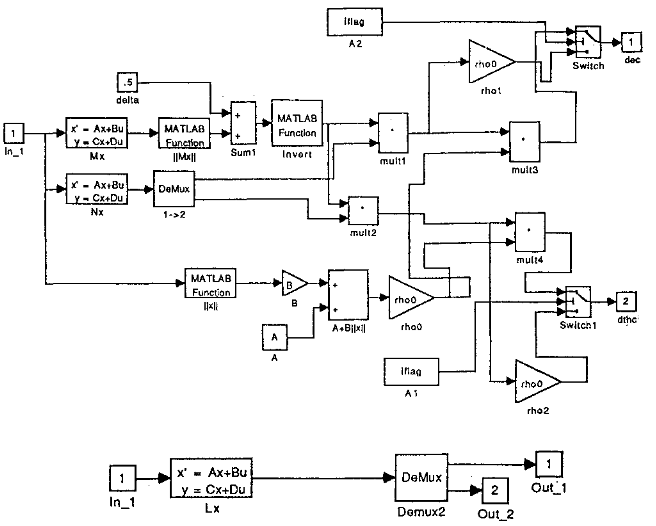

Fig. 7 Block diagram variable structure systems controller.

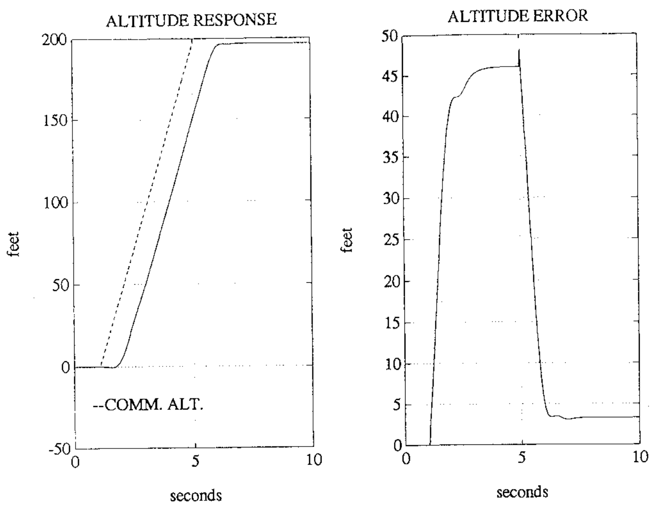

Fig. 8 Altitude response during tracking.

with the amount of uncertainty that can be tolerated by the controller, and VSS control is no exception.

Figure 11 shows altitude response and error in the presence of parameter variations ΔA_{L0} in the aircraft plant matrix. With the choice of $\Lambda_3^* = \text{diag}[-5, \ -5]$, the system tracks the commanded altitude with a maximum error of about 70 ft when 50% parameter variation is allowed. To reduce this error, which occurs because of the delay in achieving the sliding mode, more stable eigenvalues of Λ_3^* should be used, provided stability is maintained.

Figure 12 shows the altitude response and error in the presence of control effectiveness losses ΔB_{L0} of 25 and 50%. Here, $\Lambda_3^* = \text{diag}[-2, 5, \ -2.5]$ and while a 25% loss is handled by the controller, a 50% loss causes instability if the same response speed is required.

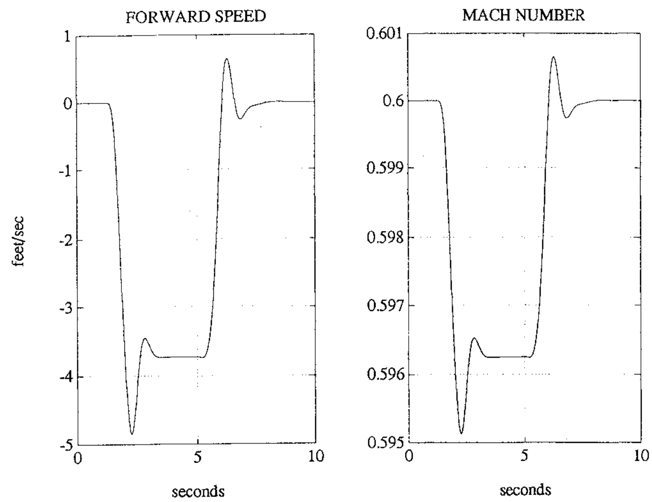

Fig. 9 Speed response during tracking.

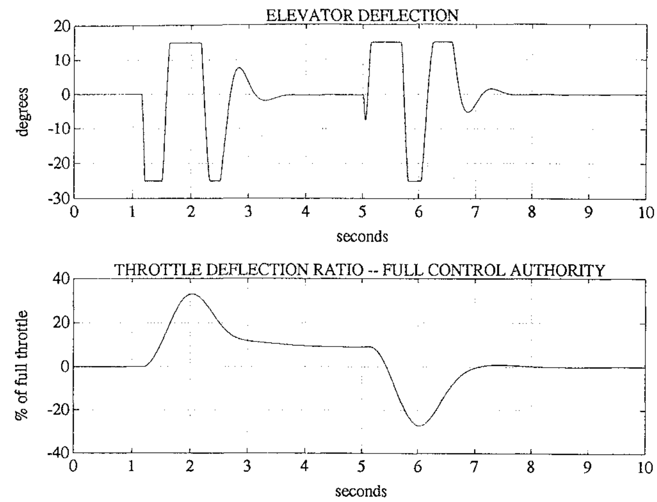

Fig. 10 Control activity during tracking.

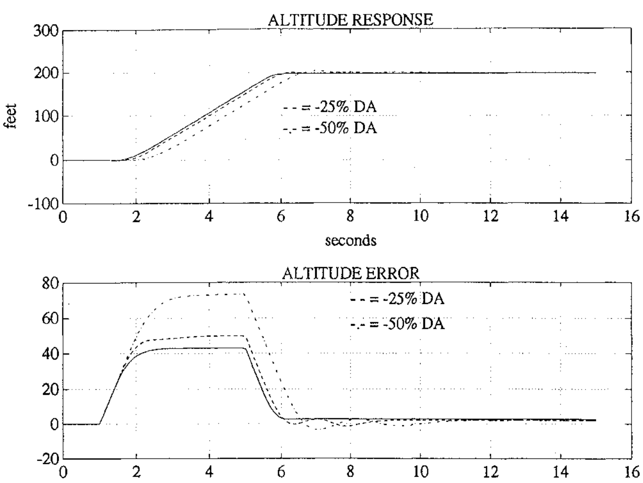

Fig. 11 Tracking in the presence of perturbations ΔA.

The behavior of the control effectors is shown in Figs. 13 and 14 respectively. With a 50% loss in effectiveness, the elevator becomes ineffective because of full saturation.

The last two figures show the system's behavior in the presence of a 50 and 75% decrease in actuator speed. As before, $\Lambda_3^* = \text{diag}[-2.5, \ -2.5]$. No loss of performance is experienced with the actuator speed reduced by half, however, a 75% loss (actuator break frequency reduced to 5 rad/s) produces a delay that causes instability, as shown in Fig. 15, with a fully saturated elevator shown in Fig. 16.

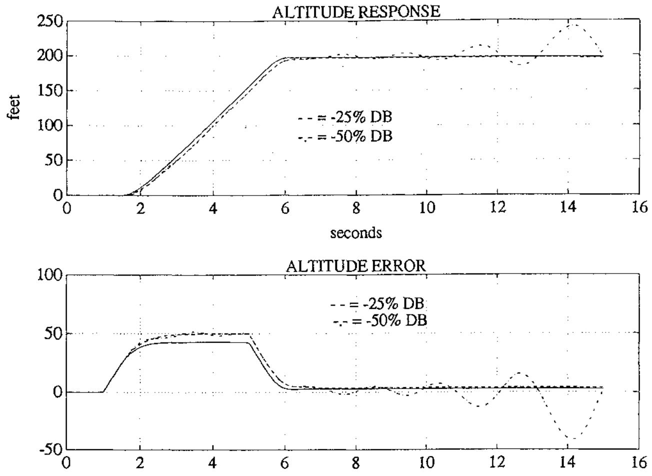

Fig. 12 Tracking in the presence of perturbations ΔB.

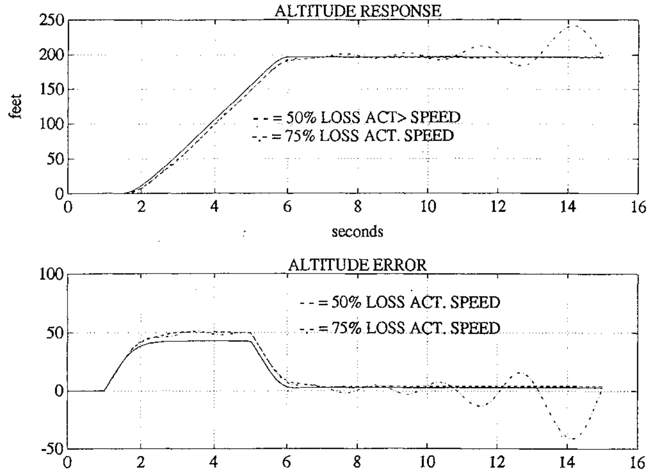

Fig. 15 Tracking in the presence of partial actuator loss.

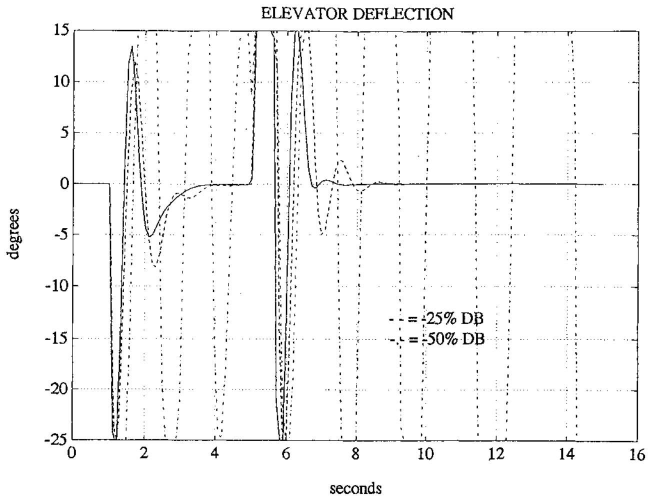

Fig. 13 Elevator activity with perturbations ΔB.

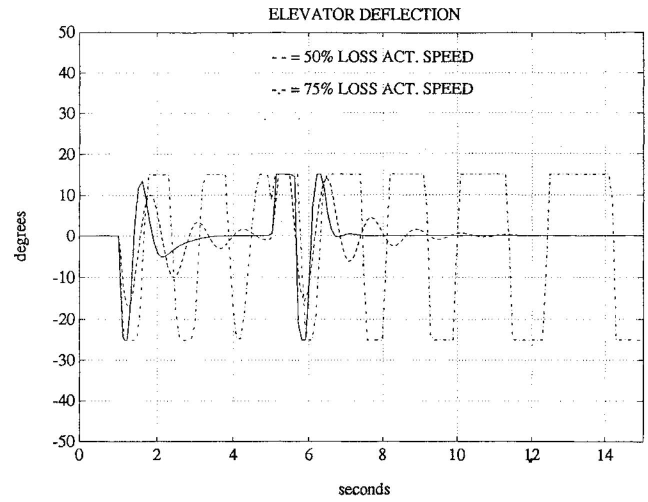

Fig. 16 Elevator activity in the presence of partial actuator loss.

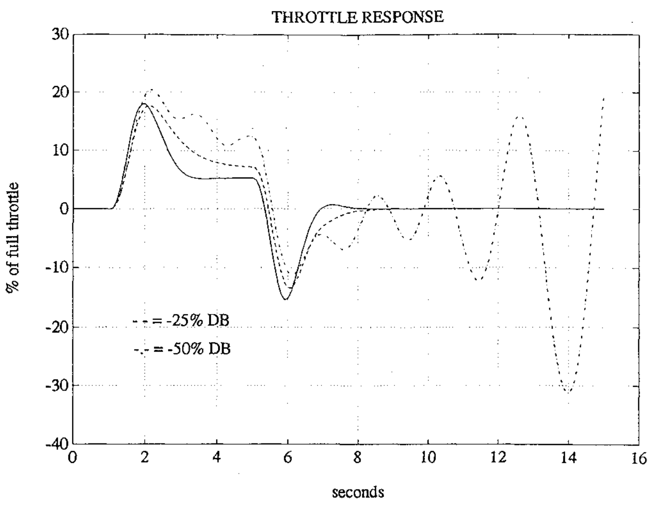

Fig. 14 Throttle activity with perturbations ΔB.

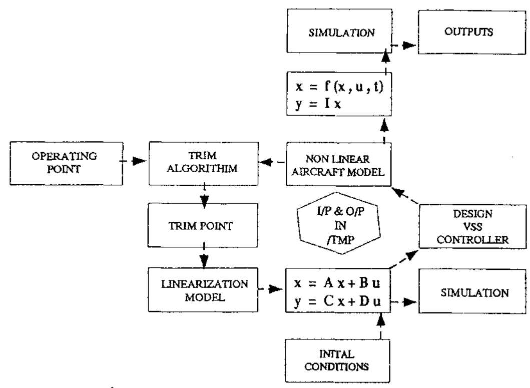

Fig. 17 Flowchart of nonlinear simulation code.

VII. Nonlinear Simulation

The nonlinear simulation code available from Ref. 1 has been converted into a modular MATLAB file by the first author. The new code has several capabilities, such as trimming the aircraft about a specific flight condition, computation of linearized models about trim, interface with the control law, and linear and nonlinear simulation of different aircraft models. A generic structure of the software, at this stage, is shown in Fig. 17. Several m-files (MATLAB specific macros) have been implemented for trimming and linearization, while the structure of the aircraft model has been maintained similar to that of the original software, the only addition being the creation of appropriate interface with MATLAB. Various controllers can be included and used with a standard call. Presently, the VSS controller derived in Example 3 is being tested, and results are not yet available. The controller will be tested for different flight conditions in its fixed gain mode, with a gain scheduler introduced if the performance is not satisfactory.

VIII. Conclusions

The present paper has presented an analysis of variable structure control as a candidate methodology for the controls design challenge. Advantages and disadvantages have been highlighted using simple numerical examples. It seems that the saturation limits imposed on the actuators by the design constraints will limit the achievable tracking characteristics. This type of control methodology has shown encouraging results, although it has not been considered a technique to be particularly acceptable in the design of flight control systems. The chattering is, in fact, eliminated, resulting in smooth control and the knowledge of uncertainties bound allows the synthesis without particular difficulties. Computer-aided packages are particularly suited for the design of the control structure and simulation, and these have been implemented on a Macintosh computer running MATLAB/SIMULINK software developed by the MathWorks, Inc.

The next stage of the design will involve several aspects. First, a more accurate model of the system is being developed. In particular, the sensor dynamics and location will be included as well as the necessary longitudinal and lateral-directional couplings. Incomplete state feedback and/or state reconstruction will then be considered. Second, alternate control structures will be tested, such as model following and conventional control that can easily be incorporated in a VSS framework. The latter essentially replaces the linear control structure used in this paper with, for example, an altitude hold or a turn coordination autopilot.

At this stage, a more accurate selection of the design parameters will be performed. Finally, discretization of the control laws will be introduced. This aspect has not been treated in detail in the VSS literature, and some methodological extensions could be developed.

Appendix

The aircraft matrices are as follows:

$$A_{LO} = \begin{bmatrix} -0.0149 & -41.8 & 0 & -32.17 & 0 \\ -0.0001 & -0.523 & 0.9985 & -0.0018 & 0 \\ 0 & -3.79 & -0.422 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -568.1 & 0 & 568.1 & 0 \end{bmatrix} \qquad B_{LO} = \begin{bmatrix} 0 & 13.71 \\ -0.045 & 0 \\ -4.712 & -0.0019 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The actuator matrices are as follows:

$$A_{ACT} = \begin{bmatrix} -20 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -20 & -21 \end{bmatrix} \qquad B_{ACT} = \begin{bmatrix} 20 & 0 \\ 0 & 0 \\ 0 & 10 \end{bmatrix}$$

The gain matrices for example 1 are as follows:

$$L = \begin{bmatrix} -0.0103 & -46.33 & 2.397 & 56.79 & 0.048 & 0.0105 & -0.1812 & -0.01415 \\ -0.0242 & 11.82 & -0.0746 & -11.84 & -0.0063 & -0.01415 & 0.2049 & 0.7628 \end{bmatrix}$$

$$M = \begin{bmatrix} 0.968 & -144.3 & 1.318 & 145.5 & 0.2515 & 0 & 5.242 & 1 \\ 0.1213 & 167.09 & -3.5894 & -188.74 & -0.484 & 0.5 & 0.1415 & 0 \end{bmatrix}$$

$$N = \begin{bmatrix} 0.006 & 8.354 & -0.179 & -9.437 & -0.0242 & 0.025 & 0.0071 & 0 \end{bmatrix}$$

The gain matrices for example 2 are as follows:

$$L = \begin{bmatrix} -0.0345 & -79.74 & 3.115 & 94.37 & 0.145 & -0.0895 & -0.2095 & 0.01415 \\ -0.145 & 29.86 & -0.239 & -30.02 & -0.0377 & -0.01415 & -0.4505 & 0.6379 \end{bmatrix}$$

$$M = \begin{bmatrix} 0.1614 & -24.05 & 0.2917 & 24.25 & 0.0419 & 0 & 0.8737 & 0.1667 \\ 0.0404 & 55.69 & -1.196 & -62.91 & -0.1613 & 0.1667 & 0.0471 & 0 \end{bmatrix}$$

$$N = \begin{bmatrix} 0.002 & 2.785 & -0.0599 & -3.145 & -0.0081 & 0.0083 & 0.00235 & 0 \\ 0.0081 & -1.203 & 0.0109 & 1.2124 & 0.0021 & 0 & 0.0437 & 0.0083 \end{bmatrix}$$

$$F_F = \begin{bmatrix} -0.1452 \\ 0.0377 \end{bmatrix}$$

The gain matrices for example 3, ΔA_{LO} perturbation are as follows:

$$L = \begin{bmatrix} -0.0587 & -133.2 & 3.833 & 132.3 & 0.242 & -0.1895 & -0.237 & -0.01415 \\ -0.242 & 44.29 & -0.3712 & -44.57 & -0.0628 & -0.01415 & -0.9747 & 0.538 \end{bmatrix}$$

$$M = \begin{bmatrix} 0.0968 & -14.43 & 0.132 & 14.55 & 0.0251 & 0 & 0.524 & 0.1 \\ 0.0243 & 33.29 & -0.7178 & -37.75 & -0.0968 & 0.1 & 0.0281 & 0 \end{bmatrix}$$

$$N = \begin{bmatrix} 0.0012 & 1.67 & -0.036 & -1.887 & -0.00484 & 0.005 & 0.0014 & 0 \\ 0.0048 & -0.722 & 0.0066 & 0.73 & 0.00125 & 0 & 0.0262 & 0.005 \end{bmatrix}$$

$$F_F = \begin{bmatrix} -0.242 \\ 0.0628 \end{bmatrix}$$

The gain matrices for example 3, ΔB_{LO}, ΔB_{ACT} perturbations are as follows:

$$L = \begin{bmatrix} -0.0285 & -71.39 & 2.936 & 85.1 & 0.121 & -0.0645 & -0.2024 & -0.0142 \\ -0.121 & 26.25 & -0.2065 & -26.39 & -0.0314 & -0.0142 & -0.3194 & 0.6629 \end{bmatrix}$$

$$M = \begin{bmatrix} 0.1936 & -28.86 & 0.2636 & 29.1 & 0.0502 & 0 & 1.05 & 0.2 \\ 0.048 & 66.84 & -1.436 & -75.5 & -0.194 & 0.2 & 0.056 & 0 \end{bmatrix}$$

$$N = \begin{bmatrix} 0.0024 & 3.342 & -0.072 & -3.775 & -0.0097 & 0.01 & 0.0028 & 0 \\ 0.0096 & -1.443 & 0.0132 & 1.455 & 0.0025 & 0 & 0.0524 & 0.01 \end{bmatrix}$$

$$F_F = \begin{bmatrix} -0.121 \\ 0.0314 \end{bmatrix}$$

References

[1]Duke, L., "Controls Design Challenge Proposal," *Proceedings of the AIAA Guidance, Navigation, and Control Conference,* AIAA, Washington, DC, 1990.

[2]Emel'yanov, S. V., "Design of Variable Structure Control Systems with Discontinuous Switching Functions," *Engineering Cybernetics* [*Teknicheskaya Kibernetica*], Vol. 1, 1964.

[3]Utkin, V. I., "Sliding Modes and their Application in Variable Structure Systems," *MIR Moscow*, 1978.

[4]Itkis, U., *Control Systems of Valuable Structure*, Wiley, New York, 1976.

[5]Drazenovic, B., "The Invariance Conditions in Variable Structure Systems," *Automatica,* Vol. 5, 1969.

[6]Zinober, A. S. I. (ed.), *Deterministic Control of Uncertain Systems*, No. 40, Institute of Electrical and Electronic Engineers, NY, Control Engineering Series, 1990.

[7]Calise, A. J., and Kramer, F. S., "A Variable Structure Approach to Robust Control of a VTOL Aircraft," *Journal of Guidance, Control, and Dynamics,* Vol. 7, No. 5, 1984.

[8]Mudge, S. K., and Patton, R. J., "Enhanced Assessment of Robustness for an Aircraft's Sliding Mode Controller," *Journal of Guidance, Control, and Dynamics,* Vol. 11, No. 6, 1988.

[9]Slotine, J-J. E., "Sliding Controller Design for Nonlinear Systems," *International Journal of Control,* Vol. 40, No. 2, 1984.

[10]Vadali, S. R., "Variable Structure Control of Spacecraft Large Angle Maneuvers," *Journal of Guidance, Control, and Dynamics,* Vol. 9, No. 2, 1986.

[11]Ryan, E P., and Corless, M., "Ultimate Boundedness and Asymptotic Stability of Uncertain Dynamic Systems via Continuous and Discontinuous Feedback Control," *IMAC. Math. Control Information,* Vol. 1, 1984.

[12]Ambrosino, G., Celentano, G., Garofalo, F., et al., "Variable Structure MRAC Systems," *International Journal of Control*, Vol. 39, 1984.

[13]Burton, J. A., and Zinober, A. S. I., "Continuous Self-Adaptive Control Using Smoothed Variable Structure Controller," *International Journal of Systems Science*, Vol. 19, 1988.

[14]Balestrino, A., DeMaria, G., Sciavicco, L., et al., "Hyperstable Adaptive Model Following Control of Nonlinear Plants," *Systems and Control Letters*, Vol. 1, 1984.

[15]Erzberger, H., "Analysis and Design of Model Following Control Systems by State Space Techniques," *Proceedings of the JACC* (Ann Arbor, MI), 1968.

[16]Dorling, C. M., Zinober, A. S. I., et al., "Two Approaches to Hyperplane Design in Multivariable Structure Control Systems," *International Journal of Control*, Vol. 44, 1986.

[17]Dorling, C. M., Zinober, A. S. I., et al., "Robust Hyperplane Design in Multivariable Structure Control Systems," *International Journal of Control*, Vol. 48, 1988.

[18]Leitmann, G., "On the Efficacy of Nonlinear Control in Uncertain Systems," *Journal of Dynamics Systems, Measurement and Control*, Vol. 102, 1981.

[19]O'Brien, M. J., and Broussard, J. R., "Feedforward Control to Track the Output of a Forced Model," *Proc. 17th CDC*, 1978.